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## LETTER TO THE EDITOR

## Casimir force on a micrometer sphere in a dip: proposal of an experiment

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### Abstract

The attractive Casimir force acting on a micrometer-sphere suspended in a spherical dip, close to the wall, is discussed. This set-up is in principle directly accessible to experiment. The sphere and the substrate are assumed to be made of the same perfectly conducting material.

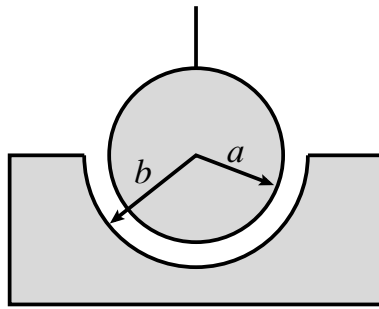
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### 1. Introduction

Experiments aiming at testing the theory of the Casimir effect ([1]; for recent reviews see [2–4]) are more numerous than what one might perhaps think. Let us here only highlight some examples, starting with the classic experiment of Sparnaay [5, 6]. This experiment tested the Casimir force between two parallel plates, made of chromium steel, chromium and aluminium. With the exception of aluminium (whose problems most likely were due to impurities), the results were in good qualitative agreement with the Lifshitz formula [7], calculated from the assumption of perfect reflecting boundaries. The experimental technique was based upon the use of a spring balance (sensitivity about  $10^{-3}$  dynes), sensing the attractive force. The plates were assigned parallel by visual inspection.

Another well-known classic experiment is the one of Sabisky and Anderson [8], dealing with the properties of liquid helium films adsorbed on cleaved surfaces of alkaline-earth fluoride crystals at  $T = 1.38$  K. Film thicknesses measured by means of acoustic interferometry were found to lie between 1 and 25 nm. At thermal equilibrium the film thickness gets a value that is determined thermodynamically, given the Lifshitz formula for the Casimir force as input. The results measured were in very good quantitative agreement with the Lifshitz expression.

The modern series of experiments was initiated with the seminal work of Lamoreaux [9]. He used a balance based on a torsion pendulum to measure the Casimir force between a gold-coated spherical lens (radius about 12 cm) and a flat plate. The lens was mounted



**Figure 1.** The proposed experimental set-up. A metal sphere of radius  $a$  is suspended in a spherically formed metallic dip of radius  $b$ .

on a piezo stack and the plate on one arm of the torsion balance. The Casimir force would result in a torque, which was detected via a capacitance measurement. Maximum separation between the two surfaces was  $12.3 \mu\text{m}$ . In a later note, Lamoreaux included corrections, such as those arising from finite conductivity [10], and with Buttler [11] he gave recently an analysis of thermal noise in torsion pendulums. The Lamoreaux experiment gave rise to a surge of activity, both experimentally and theoretically.

In the most recent years, the atomic force microscope (AFM) used in particular by Mohideen *et al* [12–14] has led to the most accurate determination of the Casimir force between a micrometer-sized sphere and a plate. By using a sphere/plate configuration, one avoids the strict requirement about parallelism that is so demanding in the case of parallel plates. The accuracy is now of the order of a few per cent; this accuracy being actually under current debate mainly because of the temperature corrections.

We shall not here go into further detail as regards the experimental status. A detailed exposition on the experiments up to 2001 is given in the review of Bordag *et al* referred to earlier [4], and a detailed survey of the developments in the last four years is given in Milton's review [2], section 3.6. We mention, though, the impressive plane-plate experiment of Bressi *et al* [15]; they were able to guarantee a parallelism of the plates to better than  $3 \times 10^{-5}$  rad. (It may even be that this experiment has been the first to measure the temperature corrections to the Casimir force; cf the discussions on temperature corrections in [16, 17].)

The main purpose of the present letter is to propose a new variant of the sphere-substrate configuration, namely a metal sphere suspended in a spherically formed metallic dip (see figure 1). We will make a simple, approximate, calculation of the vertical Casimir force on the sphere, utilizing the known theory for the Casimir effect under conditions of perfect spherical symmetry. There exist several theoretical treatments of the Casimir effect under conditions of spherical symmetry—cf [18–22] for instance—but the experimental tests of this kind of Casimir forces have so far been absent. Leaving aside practical difficulties such as the need of keeping the sphere in the dip in a stable lateral position, we hope nevertheless that the present simple idea can be of interest to experimentalists.

Temperature corrections are not included in the main formalism but are discussed in section 3. We use natural units,  $\hbar = c = 1$ , in the intermediate calculations, and we employ Heaviside–Lorentz units.

## 2. Geometrical set-up

We begin by assuming perfect spherical symmetry: let there be two concentric perfectly conducting singular shells situated at  $r = a$  and  $r = b$ . We shall be interested in the

distribution of fields in the annular region  $a < r < b$ , at  $T = 0$ . The most natural way of approach when describing this situation is to make use of the Green functions; for the case of spherical symmetry this kind of theory was worked out by Milton *et al* [18]. There occur two scalar Green functions in the problem,  $F_l(r, r')$  and  $G_l(r, r')$ . As shown in [19] for the double-shell situation, the electromagnetic boundary conditions at  $r = a, b$  transform into the following conditions for the scalar Green functions:

$$F_l(a, r') = 0, \quad \frac{d}{dr} [r G_l(r, r')]_{r=a} = 0, \quad (1)$$

and similarly for  $r = b$ .

The surface force density, here called  $f_b$ , on the outer surface  $r = b$  is calculated from Maxwell's stress tensor. To this end we need the two-point functions, which are in turn given by the components of the Green functions. Some calculation yields for the radial two-point function for the electric field

$$\langle E_r^2(b-) \rangle = \frac{1}{\pi b^4} \int_0^\infty \frac{dy}{y} \sum_{l=1}^\infty \frac{2l+1}{4\pi} l(l+1) \frac{s_l(y) - A_G(ay/b)e_l(y)}{s_l'(y) - A_G(ay/b)e_l'(y)}, \quad (2)$$

whereas the corresponding transverse function for the magnetic field becomes

$$\langle H_\perp^2(b-) \rangle = -\frac{1}{\pi b^4} \int_0^\infty y dy \sum_{l=1}^\infty \frac{2l+1}{4\pi} \left[ \frac{s_l(y) - A_G(ay/b)e_l(y)}{s_l'(y) - A_G(ay/b)e_l'(y)} + \frac{s_l'(y) - A_F(ay/b)e_l'(y)}{s_l(y) - A_F(ay/b)e_l(y)} \right]. \quad (3)$$

Here  $\langle E_r^2(b-) \rangle$  is shorthand for  $\langle E_r(r)E_r(r') \rangle_{r' \rightarrow r=b-}$ , etc;  $y$  means the nondimensional frequency  $y = \hat{\omega}b$  with  $\hat{\omega}$  being the Wick-rotated frequency, and  $s_l(x) = \sqrt{\pi x/2} I_\nu(x)$ ,  $e_l(x) = \sqrt{2x/\pi} K_\nu$  with  $\nu = l + 1/2$  are the Riccati-Bessel functions defined such that their Wronskian is  $W\{s_l, e_l\} = -1$ . Prime means derivative with respect to the whole argument. The coefficients  $A_F$  and  $A_G$  in equation (3) are

$$A_F(x) = \frac{s_l(x)}{e_l(x)}, \quad A_G(x) = \frac{s_l'(x)}{e_l'(x)}. \quad (4)$$

The reason why the integration over  $y$  runs to infinity in equation (3) is that the medium in the surfaces is assumed to be perfectly conducting (i.e., with permittivity  $\varepsilon \rightarrow \infty$ ) for all frequencies.

The other two-point functions are found to vanish,

$$\langle E_\perp^2(b-) \rangle = \langle H_r^2(b-) \rangle = 0, \quad (5)$$

so it is simple to find the surface force density at  $r = b$  via use of the Maxwell stress tensor:

$$\begin{aligned} f_b &= \frac{1}{2} [-\langle E_r^2(b-) \rangle + \langle H_\perp^2(b-) \rangle] \hat{\mathbf{r}} \\ &= \frac{-1}{2\pi b^4} \int_0^\infty y dy \sum_{l=1}^\infty \frac{2l+1}{4\pi} \left[ \frac{s_l'(y) - A_F(ay/b)e_l'(y)}{s_l(y) - A_F(ay/b)e_l(y)} + \frac{s_l''(y) - A_G(ay/b)e_l''(y)}{s_l'(y) - A_G(ay/b)e_l'(y)} \right] \hat{\mathbf{r}}. \end{aligned} \quad (6)$$

The surface force  $F_z$  that we are interested in, is the  $z$  component of the surface force density integrated over the lower hemisphere,  $\frac{1}{2}\pi < \theta < \pi, 0 < \phi < 2\pi$ . This integration is trivial, since the magnitude of the surface force density contains no angular dependence. It is moreover convenient to rewrite  $F_z$  in such a way that the mutual contribution is separated out.

Some formal manipulations yield

$$F_z = \frac{1}{2b^2} \int_0^\infty y \, dy \sum_{l=1}^{\infty} \frac{2l+1}{4\pi} \left[ \frac{s'_l(y)}{s_l(y)} + \frac{s''_l(y)}{s'_l(y)} \right] + \frac{1}{2b^2} \int_0^\infty y \, dy \sum_{l=1}^{\infty} \frac{2l+1}{4\pi} \frac{\partial}{\partial y} \ln \left[ \left( 1 - A_F(x) \frac{e_l(y)}{s_l(y)} \right) \left( 1 - A_G(x) \frac{e'_l(y)}{s'_l(y)} \right) \right]. \quad (7)$$

Here,  $x = ay/b$  is a function of  $y$ , but when taking the partial derivative with respect to  $y$  in the last term,  $x$  and  $y$  are regarded as independent variables. The first term describes the self-force on the surface  $r = b$ , due to the fluctuating fields in the annular region. (If we were taking into account the contribution from the outer region  $r > b$  also, as would strictly speaking be necessary when considering a double spherical shell, then there would be an analogous term  $e'_l/e_l + e''_l/e'_l$  in addition; cf [18].) In our case, self-forces are not of interest. The physically important force thus becomes

$$F_z = \frac{1}{2b^2} \int_0^\infty y \, dy \sum_{l=1}^{\infty} \frac{2l+1}{4\pi} \frac{\partial}{\partial y} \ln \left[ \left( 1 - A_F(x) \frac{e_l(y)}{s_l(y)} \right) \left( 1 - A_G(x) \frac{e'_l(y)}{s'_l(y)} \right) \right]. \quad (8)$$

The expression is positive, corresponding to an upward directed force on the outer wall  $r = b$ , which in turn means a downward-directed force on the sphere. The expression is finite as it stands; no regularization procedure is necessary.

To make the expression more practically useful, we employ the Debye expansion for the Riccati-Bessel functions. The calculation is parallel to that in [19], and will not be repeated here. We give the result in dimensional form, when reintroducing  $a$  instead of  $b$  and working to the first order in the small quantity  $d/a$ ,

$$F_z = \frac{\pi^2 \hbar c}{240d^4} (\pi a^2) \left( 1 + \frac{4d}{3a} \right). \quad (9)$$

Here we have separated out the standard expression  $\pi^2 \hbar c / 240d^4$  for the Casimir surface force density between flat parallel plates. It is seen from equation (9) that the curvilinear geometry leads to a slightly increased force as compared with the force between parallel plates having an effective area of  $\pi a^2$ . For instance, if  $a = 50 \, \mu\text{m}$ ,  $d = 1 \, \mu\text{m}$ , the last correction term in equation (9) amounts to 2.6%.

### 3. Discussion

The most characteristic property of the present proposal is that it suggests how the Casimir formalism worked out for spherically symmetric geometries can be exposed to an experimental test. As far as we know, this is the first proposal of such a kind. Of course, our calculation above is only approximate. Let us make a few final remarks:

- The most evident simplification that we have made is to assume that the field distribution is the same as in the case of perfect spherical symmetry, all over the dip. Of course, near  $\theta = \frac{1}{2}\pi$  there are 'stray' fields making the distribution different from the perfectly symmetric case. A circumstance, which however diminishes the influence from the stray fields, is that  $z$  component becomes only slightly influenced near  $\theta = \frac{1}{2}\pi$ . We can estimate the magnitude of the effect by performing the integral over  $\theta$  in the  $F_z$  calculation from  $\frac{1}{2}\pi + \Delta$  to  $\pi$ , instead of from  $\frac{1}{2}\pi$  to  $\pi$ . The result is that expression (9) gets multiplied with a correction factor  $(1 - \sin^2 \Delta)$ . For example, taking the error introduced by the stray fields to be  $\Delta = 5^\circ$ , we get a decrease in  $F_z$  of about 0.8%. It would be quite a

difficult task to make an accurate calculation of the influence from the stray fields; one would have to solve the complicated field distribution problem around  $\theta = \frac{1}{2}\pi$ .

- It is of interest to compare our results with the new technique proposed by Jaffe and Scardicchio based on optical paths [23–25]. This technique assumes classical optics; it is most accurate at short wavelengths where diffraction is not important. The technique has so far been applied to the case of scalar fields. The main procedure is to write the Casimir energy as a trace of the Green function; then the Green function is replaced by the sum over contributions from optical paths labelled by the number of reflections from the conducting surfaces. There are two central quantities in the analysis: first, there is the length  $l_r(x)$  of the closed geometric optics ray beginning and ending at the point  $x$  and reflecting  $r$  times from the surfaces; secondly, there is the so-called enlargement factor  $\Delta_r(x)$  of classical optics associated with the  $r$ -reflection path beginning and ending at  $x$ . Consider first the simple sphere–plate configuration. The original wavefront leaving  $x$  is spherical. The first reflection from the sphere produces a new wavefront, with in general two unequal radii of curvature. When the next incident upon the sphere, the asymmetric wavefront will be transformed in a complicated manner, not yet worked out even for the scalar field. If we now consider our proposed experimental setting where there are *two* curved surfaces, this method appears to be quite complicated. We shall therefore not try to work out this, but it is of interest nevertheless to compare our results with those obtained for the sphere–plate configuration.

Let  $f^{\text{optical}}(d/a)$  be the correction factor for a sphere and a plate, calculated by the optical method. This factor gives the ratio between the Casimir force and the force obtained for two parallel plates separated by a gap  $d$ . From [24] we have, to the first order in  $d/a$ ,

$$f^{\text{optical}}(d/a) = 1 + 0.05d/a. \quad (10)$$

We can compare this with the result calculated from the proximity force approximation (PFA) [27]. Actually there is an ambiguity in the PFA: the basic idea of the method is to apply the parallel-plate result to infinitesimal bits of the (in general) curved surfaces and integrate them up. The ambiguity lies in which surface is chosen for the integration. The physically best choice turns out to be the *plate-based* PFA, according to which

$$f_{\text{PFA}}^{\text{plate}}(d/a) = 1 - (1/2)d/a. \quad (11)$$

It is seen that even the signs in the correction terms in equations (10) and (11) are different. Now, the sphere–plate situation for the scalar field has actually been calculated numerically to a high accuracy [26]. The numerical results show clearly that the interaction energy increases with increasing values of  $d/a$ . From figure 4 in [24] it is seen that the optical approximation works well up to  $d/a \approx 0.2$ .

Finally let us compare these results with our expression (9) for the Casimir force. We see that our expression corresponds to the correction factor

$$f(d/a) = 1 + (4/3)d/a. \quad (12)$$

The experimental configuration that we have proposed in this letter is of course different from the sphere–plate configuration with scalar fields, but we see that our correction term in the factor  $f(d/a)$  is positive. There is thus a qualitative agreement between our result and the optical method result for scalar fields, equation (10), as well as with the numerical result in [26].

- We next consider the correction coming from finite temperatures. This point is quite subtle. It is instructive to start with the case where two parallel plates are separated by a

gap  $d$ . The general condition for applying  $T = 0$  theory with reasonable accuracy is that  $Td \ll 1$  (in natural units). Consider, for example, two gold plates at a gap of  $d = 1 \mu\text{m}$ ; from figure 5 in [16] it follows that the surface pressure is about 1 mPa at  $T = 300 \text{ K}$  and about 1.15 mPa at  $T = 0$ , thus a *decrease* of 15% at room temperature as compared with zero temperature. As we mentioned earlier, the parallel plate experiment of Bressi *et al* [15] may even have been able to measure this temperature effect for the case of narrow gaps,  $d \leq 0.5 \mu\text{m}$ . (In this experiment the plates were actually coated with chromium rather than with gold; when  $d = 0.5 \mu\text{m}$  corresponding to  $Td = 0.065$  at room temperature, the surface pressure is calculated to be 15.5 mPa whereas the  $T = 0$  theory yields 20.8 mPa. The theoretically predicted force reduction is thus somewhat greater than above, about 25%; cf also the discussion in [16].)

No consensus has so far been reached in the literature as regards the temperature correction, not even in the simple case of parallel plates. The essential physical point is whether the transverse electric (TE) mode contributes to the Casimir effect for a metal in the limit of zero frequency, corresponding to a Matsubara integer equal to zero. In our opinion it does *not*, as spelled out in detail in ([16]. The papers of Sernelius and Boström are in agreement with this opinion [28–32]. It implies, as mentioned, that the Casimir force is weaker (by some per cent when  $d = 1 \mu\text{m}$ ) at room temperature than at zero temperature. By contrast, the recent paper of Chen *et al* [33], which is based upon a reanalysis of the earlier atomic force microscopy experiment reported in [14], claims the temperature correction to be so small as to be negligible. In that apparatus a gold-coated polystyrene sphere, mounted on a cantilever of an AFM, was brought close to a metallic surface and the deflection of the cantilever was measured as a function of the distance. As a general remark on this experiment, in spite of the apparent excellent agreement of the experiment, one suspects that the accuracy of it has been overestimated. As discussed by Iannuzzi *et al* [34], and by Milton [2], at very short distances  $d$  (in the experiment the shortest distance was equal to 62 nm), the force at  $d = 62 \text{ nm}$  differs from the force at  $= 62 + \delta$  by more than 3.5 pN (the experimental uncertainty claimed by the authors) when  $\delta$  is larger than a few angstroms. This implies that  $d$  should have been measured with atomic precision in order to correspond to the accuracy claimed. One may compare this with the real error in the experiment: the value of  $d$  was determined with  $\pm 1 \text{ nm}$  accuracy. Moreover, the reason for the claimed smallness of the temperature correction in [33] is that the analysis is based upon the plasma dispersion relation, which in turn implies that the zero TE mode contributes to the Casimir force.

After these introductory remarks on the parallel-plate geometry, we now turn to the finite temperature version of the expression (8). It is convenient to work in terms of the free energy  $F$ , instead of the force  $F_z$ . The relationship between these quantities is  $F_z = \partial F / \partial b$  (recall that  $F_z$  is with our sign conventions positive, whereas the interaction free energy  $F$  has to be negative). The finite temperature version of  $F$  is given by

$$\beta F = \frac{1}{2} \sum_{m=0}^{\infty} \sum_{l=1}^{\infty} \nu \ln \left[ \left( 1 - A_F(x) \frac{e_l(y)}{s_l(y)} \right) \left( 1 - A_G(x) \frac{e'_l(y)}{s'_l(y)} \right) \right], \quad (13)$$

where  $\beta = 1/T$ ,  $\nu = l + 1/2$ ,  $x = 2\pi m a / \beta$  and  $y = 2\pi m b / \beta$  being the nondimensional frequencies. The prime on the  $m$  summation means that  $m = 0$  is to be taken with half-weight.

We shall consider some limiting cases of this expression. First consider the *static* case, whereby we mean that the frequency is zero ( $m = 0$ ). This means that we take the analytical limits of  $s_l(x)$  and  $e_l(x)$  when  $x \rightarrow 0$ . The result becomes (cf [22]):

$$\beta F(m = 0) = \frac{1}{2} \sum_{l=1}^{\infty} \nu \ln \left[ 1 - \left( \frac{a}{b} \right)^{2\nu} \right]. \quad (14)$$

In this formula, the radii  $a$  and  $b$  are arbitrary. The case, where this formula is most useful, is that of moderate or large gap widths. For comparison, we mention that at room temperature the Casimir force between a large ( $a = 300 \mu\text{m}$ ) gold sphere and a copper plate gets its dominant contribution from the zero frequency when  $d$  is greater than about  $0.7 \mu\text{m}$  (cf figure 3 in [35]). The physical reason is that for large  $d$  it is generally the low frequencies that become most important.

Consider the next case of finite temperatures, assuming the gap to be narrow,  $\xi \equiv d/a \ll 1$ . Then, from the uniform asymptotic (Debye) expansions for the Riccati-Bessel functions we obtain, to the first order in  $\xi$  [20]:

$$\beta F = \sum_{m=0}^{\infty} \sum_{l=1}^{\infty} \nu \ln(1 - e^{-2\xi\sqrt{\nu^2+m^2t^2}}), \quad (15)$$

where  $t = 2\pi a/\beta$  is the nondimensional temperature. This expression puts no restriction on the temperature.

Finally let us consider the case of high temperatures. Then, only the lowest Matsubara frequency  $m = 0$  contributes, and we are led back to expression (14). If we in addition require  $\xi$  to be small, we obtain [20]

$$\beta F(m = 0) = \frac{1}{8\pi} \int_0^{\infty} q \, dq \ln(1 - e^{-2qd}) = -\frac{\zeta(3)}{32\pi d^2}. \quad (16)$$

This is the same as the  $m = 0$  expression obtained for the free energy for parallel plates [35]. (Recall that when the PFA is assumed, it is this expression that is used for the force calculation for a sphere/plate system.)

Numerical calculations show in general that there is for low temperatures a flat plateau for the free energy extending up to quite high values of  $t$ . For instance, when  $d/a = 0.05$  the  $T = 0$  theory can be used up to  $\log_{10} t \approx 1.3$ , or  $t = 20$  (figure 6 in [22]). Since dimensionally  $t = 2\pi a k_B T/\hbar c$ , this corresponds to  $T \approx 140 \text{ K}$  if  $a = 50 \mu\text{m}$ .

For low and moderate temperatures, as mentioned, the Casimir force diminishes with increasing  $T$ . For comparison, when dealing with the force between a gold sphere and a copper plate, we found in [17] the  $T = 300 \text{ K}$  force to be weaker than the  $T = 0$  force by 3.6% in the case of a width  $d = 0.2 \mu\text{m}$ . This is comparable in magnitude with the experimental results of Decca *et al* [36, 37].

To conclude: it should in principle be possible to measure expression (9) for  $F_z$ , given that experimental lateral stability problems for the sphere can be overcome. One should thus expect to be able to check the predicted effective area factor of  $\pi a^2$ . A practical problem is however that the last correction factor in equation (9) appears to be of the same order of magnitude as the temperature correction.

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